WEEKLY TEST MEDICAL PLUS -01 TEST - 07 Balliwala SOLUTION Date 30-06-2019

[PHYSICS]

1. $$\text{Average speed} = \frac{\text{total distance covered}}{\text{total time taken}}$$

$$v_{av.} = \frac{\frac{x}{2}+\frac{x}{2}}{\frac{x/2}{40}+\frac{x/2}{60}} = \frac{x}{\left(\frac{x}{80}+\frac{x}{120}\right)}$$

$$= \frac{80\times120}{(120+80)} = 48\ \text{km/h}$$

2. $200 = u \times 2 - (1/2)\,a(2)^2$ or $u - a = 100$ ....(i)

$200 + 220 = u(2 + 4) - (1/2)\,(2 + 4)^2 a$

or $u - 3a = 70$ .....(ii)

Solving eqns. (i) and (ii), we get; $a = 15\ \text{cm/s}^2$ and $u = 115$ cm/s.

Further, $v = u - at = 115 - 15 \times 7 = 10$ cm/sec.

3. When a body slides on an inclined plane, component of weight along the plane produces an acceleration

$$a = \frac{mg\sin\theta}{m} = g\sin\theta = \text{constt.}$$

If s be the length of the inclined plane, then

$$s = 0 + \frac{1}{2}at^2 = \frac{1}{2}g\sin\theta \times t^2$$

$$\therefore \quad \frac{s'}{s} = \frac{t'^2}{t^2} \text{ or } \frac{s}{s'} = \frac{t^2}{t'^2}$$

Given t = 4 sec and $s' = \frac{s}{4}$

$$\therefore \quad t' = t\sqrt{\frac{s'}{s}} = 4\sqrt{\frac{s}{4s}} = \frac{4}{2} = 2\,\text{sec}$$

4. Given that; a = 3t + 4 or $\frac{dv}{dt} = 3t + 4$

$$\therefore \quad \int_0^v dv = \int_0^t (3t+4)dt \text{ or } v = \frac{3}{2}t^2 + 4t$$

$$v = \frac{3}{2}(2)^2 + 4(2) = 14\ \text{ms}^{-1}$$

5. **For first body** :

$$\frac{1}{2}gt^2 = 176.4 \quad \text{or} \quad t = \sqrt{\frac{176.4\times2}{10}}$$

or $t = 5.9$ s

For second body : t = 3.9 s

$$u(3.9)+\frac{1}{2}g(3.9)^2=176.4$$

$$3.9u+\frac{10}{2}(3.9)^2=176.4$$

or u = 24.5 m/s

6. The resultant velocity of the boat and river is 1.0 km/0.25 h = 4 km/h.

Velocity of the rive $=\sqrt{5^2-4^2}=3\ \text{km/h}$

7. Let he be the height of the tower.
Using $v^2 – u^2 = 2as$, we get;
Here, u = u, a = – g, s = – h and v = – 3u (upward direction + ve)
∴ $9u^2 – u^2 = 2gh$ or $h = 4u^2/g$

8. $$t=\sqrt{\frac{2h}{g}}$$

$$s=10\times\frac{t}{2}-\frac{1}{2}g\times\frac{t^2}{4}=5\sqrt{\frac{2h}{g}}-\frac{g}{8}\frac{2h}{g}$$

$v^2 – u^2 = 2gh$ or $100 = 2gh$ or $10=\sqrt{2gh}$

$$s=\sqrt{\frac{2gh\times 2h}{4\times g}}-\frac{h}{4}=h-\frac{h}{4}=\frac{3h}{4}$$

9. $$t=\frac{1}{u+v}=\frac{1}{\frac{l}{t_1}+\frac{l}{t_2}}$$

or $\frac{1}{t}+\frac{1}{t_1}+\frac{1}{t_2}$ or $t=\frac{t_1t_2}{(t_1+t_2)}$

10. **For first body** :
$v^2 = u^2 + 2gh$ or $(3)^2 = 0 + 2 \times 9.8 \times h$

or $h=\frac{(3)^2}{2\times 9.8}=0.46$ m

For second body :
$v^2 = (4)^2 + 2 \times 9.8 \times 0.46$

∴ $v=\sqrt{(4)^2+(2\times 9.8\times 0.46)}=5$ m/s

11. Given y = 0
Distance travelled in 10 s,

$$S_1=\frac{1}{2}a\times 10^2=50a$$

Distance travelled in 20 s,

$$S_2=\frac{1}{2}a\times 20^2=200a$$

∴ $S_2 = 4S_1$

12. During the first 5 seconds of the motion, the acceleration is – ve and during the next 5 seconds it becomes positive. (Example : a stone thrown upwards, coming to momentary rest at the highest point). The distance covered remains same during the two intervals of time.

13. Gain in angular KE = loss in PE

If l = length of the pole, moment of inertial of the pole about the edge $=M\left[\frac{l^2}{12}+\frac{l^2}{4}\right]=\frac{Ml^2}{3}$

Loss in potential energy $= \frac{Mgl}{2}$

Gain in angular $KE = \frac{1}{2}I\omega^2 = \frac{1}{2} \times \frac{Ml^2}{3} \times \omega^2$

$\therefore \quad \frac{1}{2}\frac{Ml}{3}\omega^2 = \frac{Mgl}{2}$ or $(l\omega)^2 = 3gl$

or $l\omega = v = \sqrt{3gl}$

$= \sqrt{3 \times 10 \times 30} = 30ms^{-1}$

14. Let the velocity of the scooter be v ms^{-1}. Then $(v - 10)100 = 100$ or $v = 20\ ms^{-1}$

15. Let x be the distance between the particles after t second. Then

$x = vt - \frac{1}{2}at^2$ .....(i)

For x to be maximum,

$\frac{dx}{dt} = 0$

or $v - at = 0$

or $t = \frac{v}{a}$

Putting this value in eqn. (i), we get;

$x = v\left(\frac{v}{a}\right) - \frac{1}{2}a\left(\frac{v}{a}\right)^2 = \frac{v^2}{2a}$

16. $\frac{dv}{dt} = a$

or $\int_{v_1}^{v_2} dv = \int a\ dt$

$\therefore$ Δv = Area under a – t graph,
where, Δv = magnitude of change in velocity.

17. $-s = ut_1 - \frac{1}{2}gt_1^2$ ......(i)

$-s = -ut_3 - \frac{1}{2}gt_3^2$ .....(ii)

$-s = -\frac{1}{2}gt_2^2$ .....(iii)

$-st_3 = ut_1t_3 - \frac{1}{2}gt_1^2t_3$ ....(iv)

$-st_1 = -ut_1t_3 - \frac{1}{2}gt_3^2t_1$ ...... (v)

Adding, $-s(t_1 + t_3) = -\frac{1}{2}gt_3t_1(t_3 + t_1)$ .... (v)

Adding, $-s(t_1 + t_3) = -\frac{1}{2}gt_3t_1(t_3 + t_1)$

or $s = +\frac{1}{2}gt_2t_1$ ...... (vi)

From equns. (iii) and (vi),

$\frac{1}{2}gt_3t_1 = \frac{1}{2}gt_2^2$

∴ $t_2 = \sqrt{t_3 t_1}$

18. u = 0, a = 2 m/s², t = 10 sec

∴ $s_1 = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2 \times 100$

= 100 m

Velocity after 10 sec,

v = u + at = 0 + 2 × 10 = 20 m/s

∴ $s_2 = v \times 30 = 20 \times 30 = 600$ m

Final velocity = 0, a = – 4 m/s²

∴ $0 = v^2 + 2as_3$

$0 = (20)^2 - 2 \times 4 \times s_3$

∴ $s_3 = \frac{400}{8} = 50$ m

19. Displacement in horizontal direction = πR = πm. Displacement in vertical direction = 2R = 2 m.

∴ Resultant displacement $= \sqrt{\pi^2 + 4}$ m

20. $\frac{d\vec{v}}{dt} = \vec{a} = \frac{\vec{F}}{m} = \left(\frac{6t^2\hat{i} + 4t\hat{j}}{3}\right) m/s^2$

∴ $\vec{v} = \int_0^3 \left(\frac{6t^2}{3}\hat{i} + \frac{4t}{3}\hat{j}\right)dt$

$= \left[\frac{6t^3}{9}\hat{i} + \frac{4t^2}{6} - \hat{j}\right]_0^3 = 18\hat{i} + 6\hat{j}$

21. We know that the speed of an object, falling freely under gravity, depends only upon its height from which it is allowed to fall and not upon its mass. Since, the paths are frictionless and all the objects are falling through the same vertical height, therefore their speeds on reaching the ground must be same or ratio of their speeds = 1 : 1 : 1

22. $x = \alpha t^3 + \beta t^2 + \gamma t + \delta$

$v = \text{velocity} = \frac{dx}{dt}$

$3\alpha t^2 + 2\beta t + \gamma$

v_0 = Initial velocity (at t = 0) = γ

Similarly, acceleration

$a = \frac{dv}{dt} = 6\alpha t = 2\beta$

Initial acceleration when t = 0

$a_0 = 2\beta$

∴ $\frac{a_0}{v_0} = \frac{2\beta}{\gamma}$

i.e., $\frac{a_0}{v_0} \propto \frac{\beta}{\gamma}$

23. Time interval of each ball thrown (t) = 2 sec and acceleration due to gravity (g) = 9.8 m/s²

We know that tiem of flight of first ball $(T) = \frac{2u}{g}$

Since, more than two balls remain in the sky, therefore time of flight of first ball (T) must be greater than 2t = 2 × 2 = 4 sec

$\frac{2u}{g} > 4$ or u > 2g = 2 × 9.8 = 19.6 m/s

24. Velocity of the thief's car with respect to ground is, v_{TG} = 10 m/s
Velocity of police man with respect to ground = v_{PG} = 5 m/s
Velocity of bullet fired by police man with respect to ground,

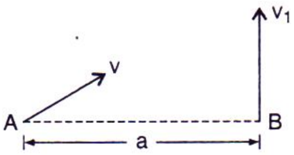
$$v_{BP} = 72\text{km/h} = \frac{72\times 5}{18} = 20\text{ m/s}$$

Velocity with which bullet will hit the target is,
$v_{BT} = v_{BG} + v_{GT}$
$= v_{BP} + v_{PG} + v_{GT}$
= 20 + 5 – 10 = 15 m/s.

25. $t = \frac{a}{u'}$

$$= \frac{a}{\sqrt{v^2 - v_1^2}}$$

$$= \sqrt{\frac{a^2}{v^2 - v_1^2}}$$



26. The nature of the path is decided by the velocity acceleration and the direction of acceleration. The trajectory can be a straight line, circle or a parabola depending on these factors.

27. As $v^2 = u^2 + 2as$
∴ $u^2 \propto s$ .....(i)
For given condition:
$u'^2 \propto 3s$ ....(ii)
From equations (i) and (ii),

$$\frac{u'^2}{u^2} = 3 \quad \text{or} \quad u' = \sqrt{3}\,v_0 \quad (\because u = v_0)$$

28. $x = 40 + 12t - t^3$
∴ Velocity, $v = \frac{dx}{dt} = 12 - 3t^2$
When particle comes to rest,

$$\frac{dx}{dt} = v = 0$$

∴ $13 - 3t^0 = 0$
or $3t^2 = 13$ or t = 2 sec
Distance travelled by the particle before coming to rest:

$$\int_0^s ds = \int_0^2 v\,dt$$

$$\therefore\ S = \int_0^2 (12 - 3t^2)dt = \left[12t - \frac{3t^3}{3}\right]_0^2$$

= 12 × 2 – 8 = 16 m

29. Time taken by a body to fall a height h to reach the ground is,

$$t = \sqrt{\frac{2h}{g}}$$

$$\therefore\ \frac{t_A}{t_B} = \frac{\sqrt{2h_A/g}}{\sqrt{2h_B/g}} = \sqrt{\frac{h_A}{h_B}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

30. When two spheres are dropped from rest, force of attraction due to the earth acts on it. Under the force of attraction they will acquire the same acceleration which is due to gravitational effect.

Also, $g = \frac{GM}{R^2}$,

in which acceleration due to gravity is independent of mass of the body. Hence, the spheres have the same acceleration.

31. The velocity upstream is (3 – 2) km/hr and down the stream is (3 + 2)km/hr.

$\therefore$ Total time taken $= \dfrac{2\text{ km}}{1\text{ km/hr}} + \dfrac{2\text{km}}{5\text{km/hr}} = 2.4$ hrs

32. $v^2 - u^2 = 2as$ or $6^2 - u^2 = 2a \times 5$
and $8^2 - u^2 = 2a(5 + 7) = 2a \times 12$
solving, $a = 2\ m/s^2$ and $u = 4$ m/s

33. $a = \dfrac{dv}{dt} = 6t + 5$

or $dv = (6t + 5)dt$
Integrating it, we have;

$$\int_0^v dv = \int_0^t (6t+5)dt$$

$\therefore$ $v = \dfrac{6t^2}{2} + 5t + C$

(where C is constant of integration)
where t = 0, v = 0, so C = 0

$\therefore$ $v = \dfrac{ds}{dt} = 3t^2 + 5t$

or $ds = (3t^2 + 5t)dt$
Integrating it within conditions of motion, i.e., as t changes from 0 to 2s, s changes from 0 to s, we have;

$$\int_0^s ds = \int_0^2 (3t^2 + 5t)dt$$

$\therefore$ $\int_0^s ds = \int_0^2 (3t^2 + 5t)dt$

34.

35.

36. **Given** : At time t = 0, velocity, v = 0

Acceleration, $f = f_0\left(1 - \dfrac{t}{T}\right)$

At f = 0, $0 = f_0\left(1 - \dfrac{t}{T}\right)$

Since, f_0 is a constant, $\therefore 1 - \dfrac{t}{T} = 0$ or t = T

Also, acceleration, $f = \dfrac{dv}{dt}$

$\therefore$ $\int_0^{v_x} dv = \int_{t=0}^{t=T} f dt = \int_0^T f_0\left(1 - \dfrac{t}{T}\right)dt$

$$v_x = \left[f_0 t - \frac{f_0 t^2}{2T}\right]_0^T = f_0 T - \frac{f_0 T^2}{2T} = \frac{1}{2} f_0 T$$

37. **Given** : $x = 9t^2 - t^3$ ......(i)

Speed, $v = \dfrac{dx}{dt} = \dfrac{d}{ct}(9t^2 - t^3) = 18t - 3t^2$

For maximum speed, $\dfrac{dv}{dt} = 0$ or $18 - 6t = 0$

$\therefore$ t = 3s

$\therefore$ $x_{max} = 81m - 27\ m = 54$ m [from eqn. (i)]

38.

39.

40. Distance travelled in the 3rd second = Distance travelled in 3s-distance travelled in 2s

As $u = 0$

$$S_{(3^{rd}s)} = \frac{1}{2}a.3^2 - \frac{1}{2}a.2^2 = \frac{1}{2}a.5$$

As $a = \frac{4}{3}ms^{-2}$

Hence, $S_{(3^{rd}s)} = \frac{1}{2}\times\frac{4}{3}\times 5 = \frac{10}{3}m$

41. $v^2 - u^2 = 2as$

Given : $v = 2\ ms^{-1}$, $u = 10\ ms^{-1}$ and $s = 135$ m

$\therefore\ a = \frac{400-100}{2\times135}$

$= \frac{300}{270} = \frac{10}{9}m/s^2$

$v = u + at$ or $t = \frac{v-u}{a} = \frac{10\ m/s}{\frac{10}{9}m/s^2} = 9$

42. Distance, $x = (t+5)^{-1}$ .....(i)

Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}(t+5)^{-1}$

$= -(t+5)^{-2}$ ....(ii)

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}[-(t+5)^{-2}]$

$= 2(t+5)^{-3}$ ....(iii)

From equation (ii), we get,

$v^{3/2} = -(t+5)^{-3}$

Substituting this in equation (iii), we get,

acceleration, $a = -2v^{3/2}$

or $a \propto (\text{velocity})^{3/2}$

From equation (i), we get,

$x^3 = (t+5)^{-3}$

Substituting this in equation (iii), we get,

Acceleration, $a = 2x^3$

or $a \propto (\text{distance})^3$ or $a \propto v^{3/2}$

[From eqn. (iv)]

Hence, option (a) is correct

43. At time t = 0, the position vector of the particle is $\vec{r}_1 = 2\hat{i} + 3\hat{j}$

At time t = 5s, the position vector of the particle is $\vec{r}_2 = 13\hat{i} + 14\hat{j}$

Displacement from $\vec{r}_1$ to $\vec{r}_2$ is

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (13\hat{i} + 14\hat{j}) - (2\hat{i} + 3\hat{j}) = 11\hat{i} + 11\hat{j}$$

$\therefore$ Average velocity

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{11\hat{i} + 11\hat{j}}{5-0} = \frac{11}{5}(\hat{i} + \hat{j})$$

44. $V(x) = bx^{-2n}$

$$a = V\frac{dV}{dx} = bx^{-2n}\{b(-2n)x^{-2n-1}\}$$

$= -2b^2nx^{-4n-1}$

45. $v = At + Bt^2$

or $\frac{dx}{dt} = At + Bt^2$

or $dx = (At + Bt^2)dt$

or $x = \left[\frac{At^2}{2} + \frac{Bt^3}{3}\right]_1^2$

$$= \frac{A}{2}(4-1) + \frac{B}{3}(8-1)$$

$$= \frac{3}{2}A + \frac{7}{3}B$$